HIGH POWER RING DESIGN STUDIES FOR SUB-CRITICAL REACTOR APPLICATION:

September 2014

MALEK HAJ TAHAR
Collider Accelerator Department

BROOKHAVEN NATIONAL LABORATORY

Overview:

- Motivation: why Acclerator Driven Sub-critical Reactor?
- Beam requirements
- Zgoubi/OPAL connection
- Analytical model «CYCLOTRON»
- Application to the case of the PSI cyclotron:Fit and Comparison with the fieldmap results.

Motivation:

• The recycling of Plutonium, Minor Actinides and Long Lived Fission Products in various types of fission reactors, Accelerator Driven Sub-critical Reactors (ADS-R) and molten salt reactors is under investigation in several countries.

• The advantages of these concepts are low waste production, high transmutation capability, enhanced safety and better resource utilization.

• Fuel removed from a reactor contains: - Minor Actinides(${}_{93}Np$, ${}_{95}Am$, ${}_{96}Cm$)
- Major Actinides (${}_{92}U$ ${}_{94}Pu$)
- Fission products(from the fission of U and Pu among which LLFP))

Why is it important to eliminate Minor Actinides?

Why Minor Actinides:

- Why is it important to eliminate MA?

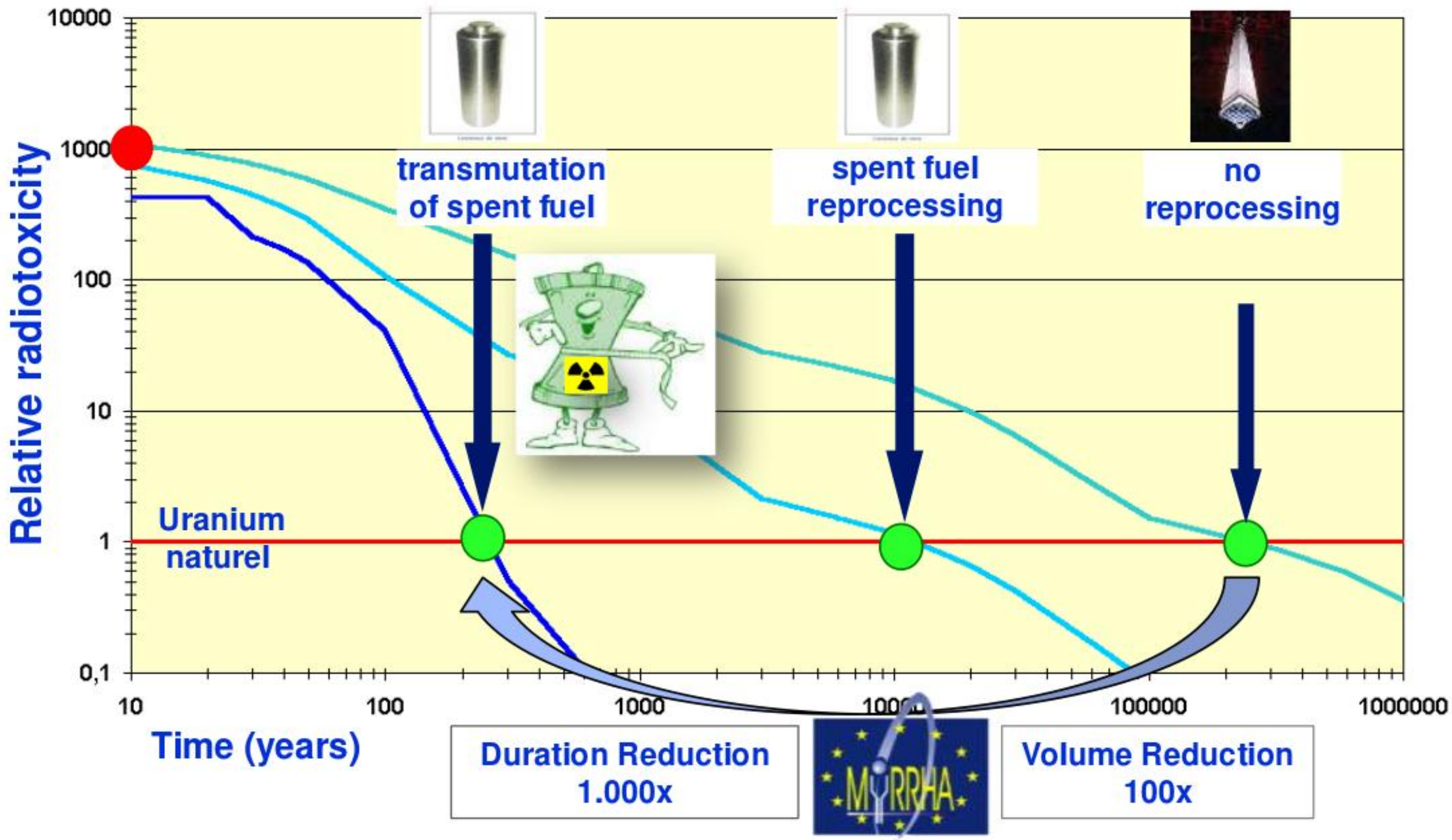
« The minor actinides are the main contributors to the heat released from vitrified waste packages, which to a large extent determines the design of repository disposal cells: the lower the heat load in the waste, the higher the disposal density of the repository». **[1]**

After three centuries, nearly 99% of the residual radiotoxicity of the vitrified wasteforms (currently produced in France) will be due to the presence of Am, Cm and their decay products.

Romoving the minor actinides from the final waste (and providing a prior storage period of about 100 years, to allow radioactive decay of the short lived fission products), could thus significantly reduce the size of the repository.

[1] Report on Sustainable Radioactive Waste Management, CEA

Why Minor Actinides:



Courtesy of the MYRRHA team

Why an accelerator?

- The major difference among the various fuels is that with each fuel a different fraction of neutrons is delayed (not produced immediately in fission).

- In the case where the fuel is mainly composed of long lived transuranic elements, it lacks Uranium’s self-regulating properties such as delayed neutrons and Doppler reactivity coefficient (even near zero).

- Because of that, it would be very difficult to control a reactor with these properties in critical regime.

⇨ Sub-critical reactor: the accelerator provides a control mechanism for this system which is by far more convenient than control rods in critical reactors.

Now let’s define the beam requirements for the accelerator.

Beam requirements (1/3)

The sub-critical core is mainly characterized by two parameters:

- The neutron effective multiplication factor defined as:

$$k_{eff} = \frac{number\ of\ neutrons\ in\ one\ generation}{number\ of\ neutrons\ in\ previous\ generation}$$

- The average number of neutrons produced per fission ν. The contribution of the reactions (n,2n) cannot be neglected especially for high energy neutrons.

We can easily derive the formula relating the thermal power of the sub-critical core to the beam parameters:

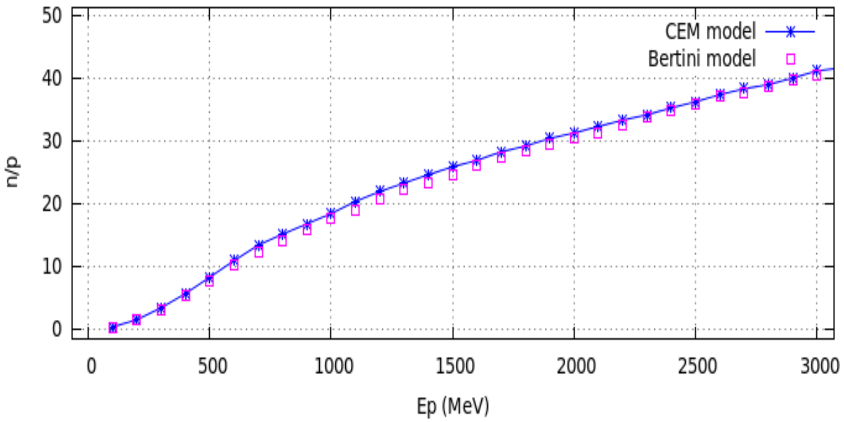
$$P_{th}(MW) = E_f(MeV) \times I(A) \times \frac{\nu_s}{\nu} \times \frac{k_{eff}}{1 - k_{eff}}$$

$E_f(MeV) \sim 200\ MeV$; ν is the number of neutrons produced per fission (~2-3)

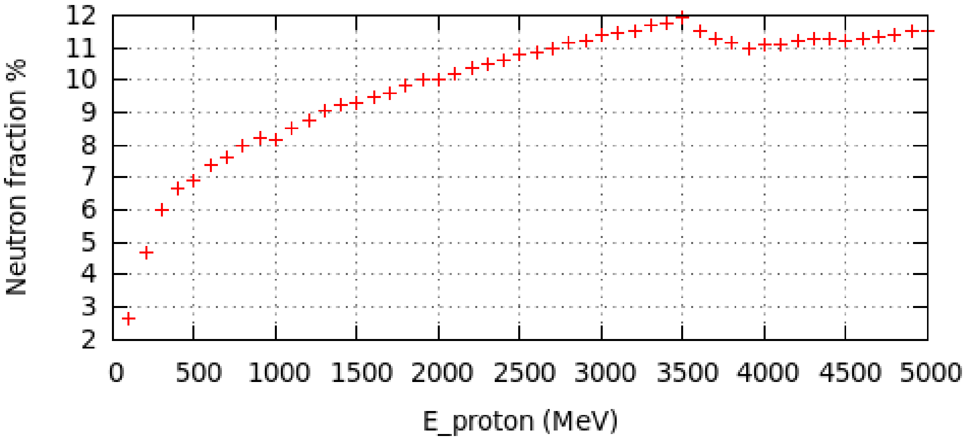
I(A) is the average intensity of the beam; ν_s is the number of neutrons injected in the core per incident proton

Beam requirements (2/3)

What's the optimal energy?



Neutron multiplicity for a lead target as a function of beam energy (MCNP6 simulations)



Fraction of neutrons from the total spectrum with energies higher than 20 MeV

Neutrons emitted in the same direction as the beam will have energies nearly as high as the incident protons. These high energy neutrons will potentially pass through the bottom shield and end up in the surrounding environment. Ground water contamination!

Beam requirements (3/3)

<u>What's the optimal current?</u>

Apply the previous formula: $$P_{th}(MW) = E_f(MeV) \times I(A) \times \frac{\mathrm{v}_s}{\mathrm{v}} \times \frac{k_{eff}}{1 - k_{eff}}$$

k_{eff} has to be fixed in such a way as to accommodate any possible positive reactivity insertion during the operation including the fuel loading stage ($k_{eff} \sim$ 0.98): <u>accounting for all known reactivity accidents</u> (core compaction, ejection of the control rods, coolant voiding, ..), MYRRHA features $k_{eff} \sim$ 0.95 which should be the maximum multiplication factor of the core.

Thus, with $\mathrm{E_p} = 1\mathrm{GeV}$ ⇨ $\mathrm{v}_s \sim 20$

$P_{th} = 400\ \mathrm{MW_{th}}$, $\mathrm{v} \sim 2.5$, $k_{eff} = 0.95$ and $E_f \sim 200\ MeV$

we obtain, $\mathrm{I} \sim 13\ mA$

NB: If you apply the previous formula with the current parameters of the MYRRHA project ($\mathrm{v}_s \sim 13$) you will obtain $\mathrm{I} \sim 3.7\ mA$ which is not far from the 3.5 mA they require.

Summary of beam requirements:

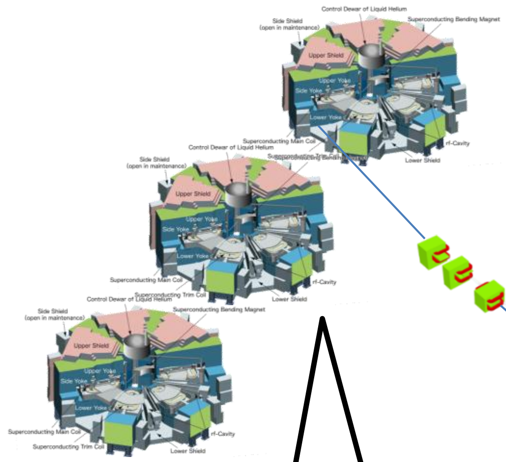
Proton beam property	Specification	Comments
Energy	[0.5, 1] GeV	
Intensity	[>26,>13] mA	Assuming P_{th}= 400 MWth
Footprint on target	Rectangular/circular	Uniformization of the beam?
Beam trip limits	Less than 10 per 3 months (exceeding 3sec) "MYRRHA"	High beam availability required
…		

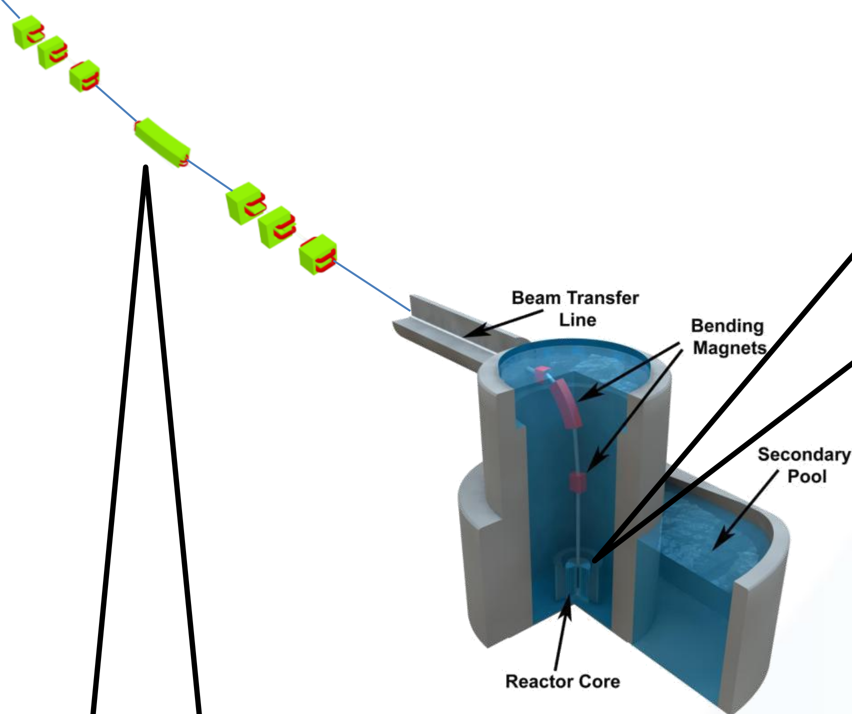
Intuitively, one expects that for such high intensities, repulsive forces between the beam particles will lead to defocusing.

For that reason we decided to use OPAL for space charge simulations.

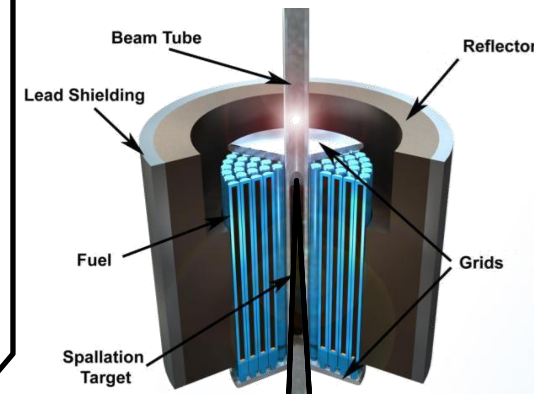
We are also focusing on Fixed Field ring methods.

LDRD proposal



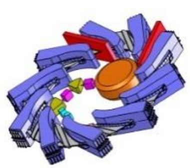


Target & Blanket

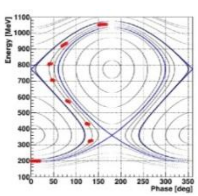


Accelerator

Multiple-ring



FFAG technology

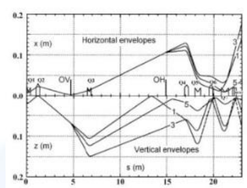


Serpentine/ CW methods

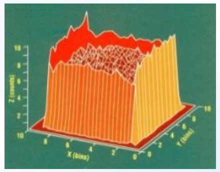
Beam transport

Space charge

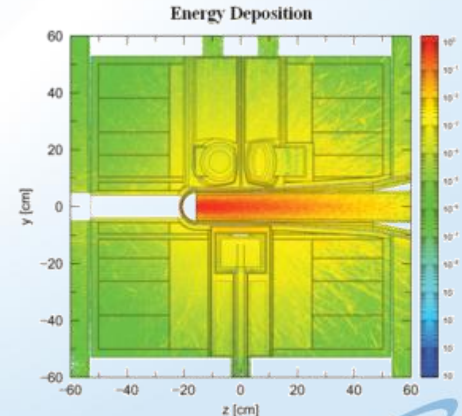
Halo control



Beam uniformization

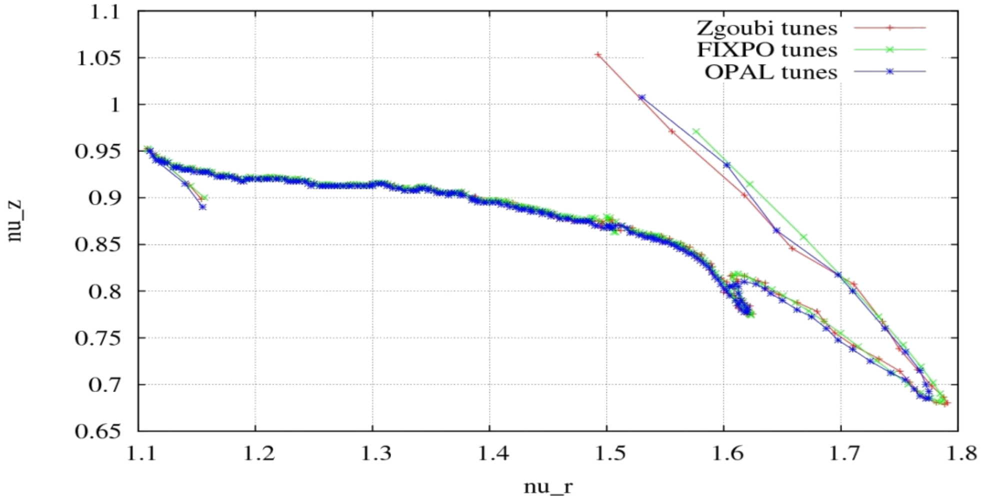


Neutronics & data

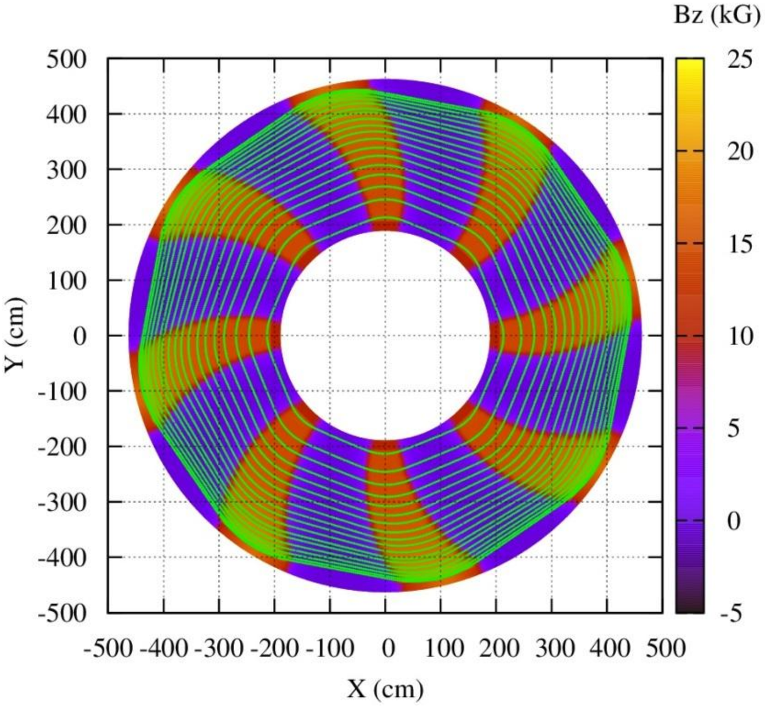


Zgoubi-OPAL connection:

Using the measured median plane field map of the PSI ring machine, the idea is to re-check the tunes and the first order linear optics: more details during the zgoubi and OPAL miniworkshop: http://www.bnl.gov/ffag14/tutorial/

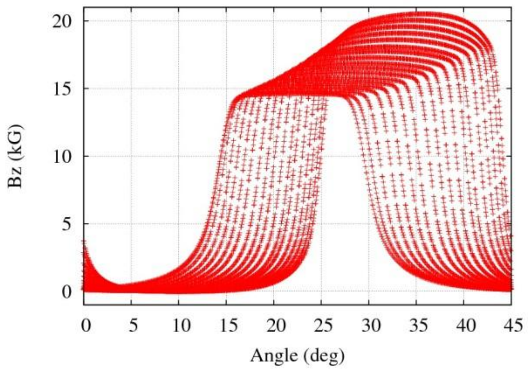


Tune diagram of the PSI ring cyclotron.



Plot of the closed orbits as well as the 2D median plane measured field map.

Field along trajectories



New element "CYCLOTRON" in zgoubi:

"**CYCLOTRON**" provides a model of a dipole field. The field along the particle's trajectory is computed in the median plane as the particle motion proceeds, by using the magnet's geometrical boundaries (according to the Enge model):

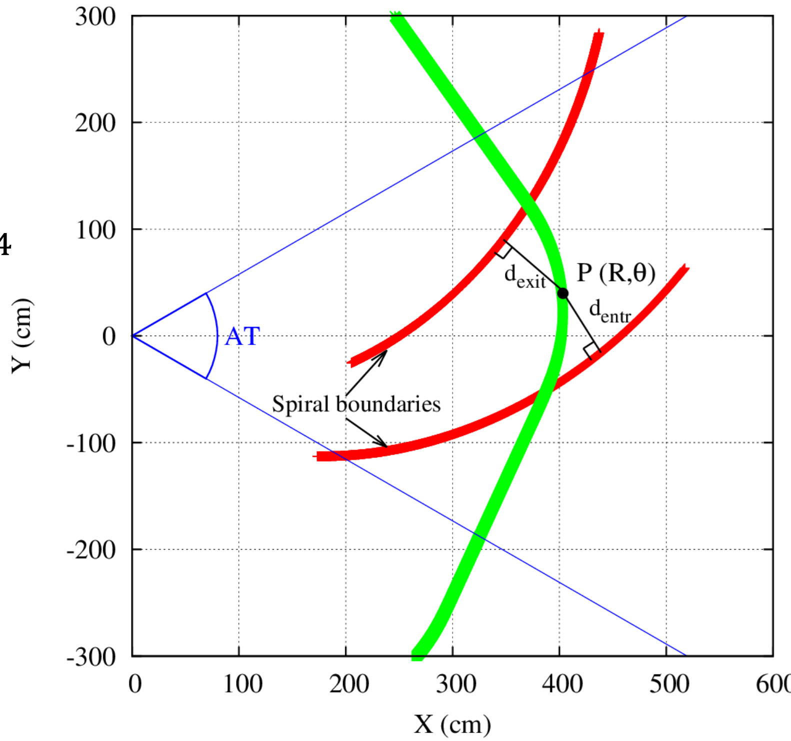
$$\boxed{B_z(R,\theta) = B_{norm} \times \mathcal{F}(R,\theta) \times \mathcal{R}(R)}$$

where

- $\mathcal{R}(\mathrm{R}) = \mathrm{B}_0 + \mathrm{B}_1 \times \mathrm{R} + \mathrm{B}_2 \times R^2 + B_3 \times \mathrm{R}^3 + \mathrm{B}_4 \times \mathrm{R}^4$

- $\mathcal{F}(R,\theta) = \mathcal{F}_{entr}(R,\theta) \times \mathcal{F}_{exit}(R,\theta)$
$$= \frac{1}{1+\exp(P_{entr}(d_{entr}))} \times \frac{1}{1+\exp(P_{exit}(d_{exit}))}$$

- $P(d) = C_0 + C_1\left(\frac{d}{gap}\right) + C_2\left(\frac{d}{gap}\right)^2 + \cdots + C_6\left(\frac{d}{gap}\right)^6$



New element "CYCLOTRON" in zgoubi:

• The effective field boundaries are modelled by a logarithmic spiral for which the angle ξ is allowed to increase radially, namely: $R = R_0 \times \exp(\frac{\theta+\omega}{\tan \xi(r)})$

where $$\xi\,(\mathrm{R}) = \xi_0 + \xi_1 \times \mathrm{R} + \xi_2 \times R^2 + \xi_3 \times \mathrm{R}^3$$

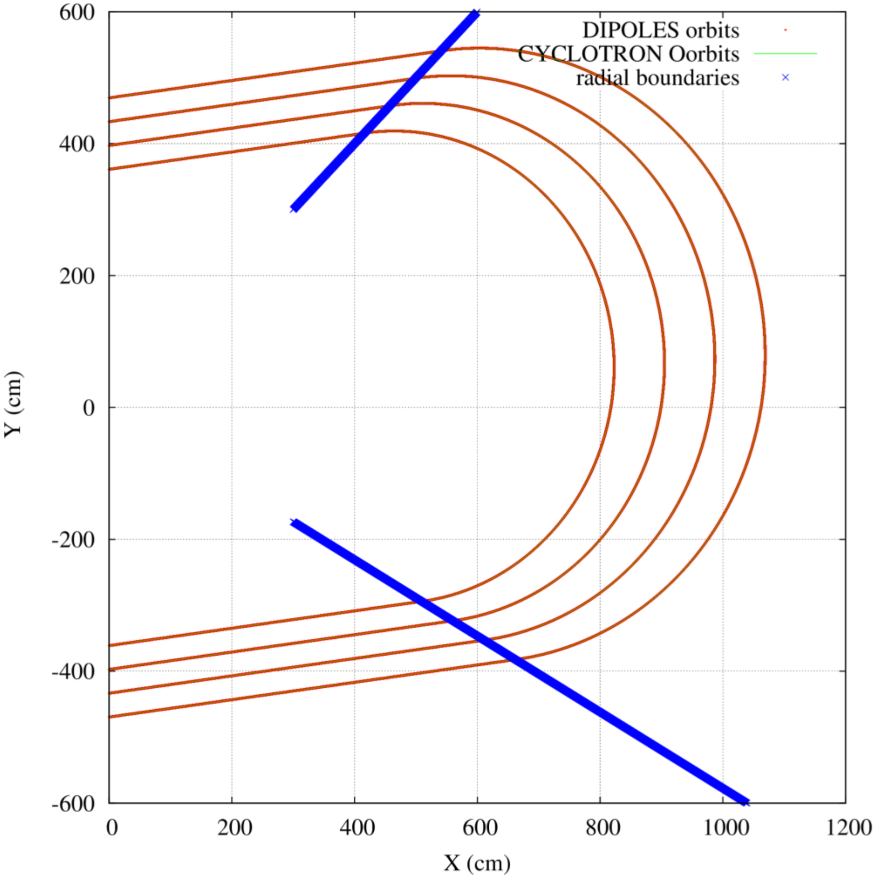
• According to this model, the gap is also allowed to vary:

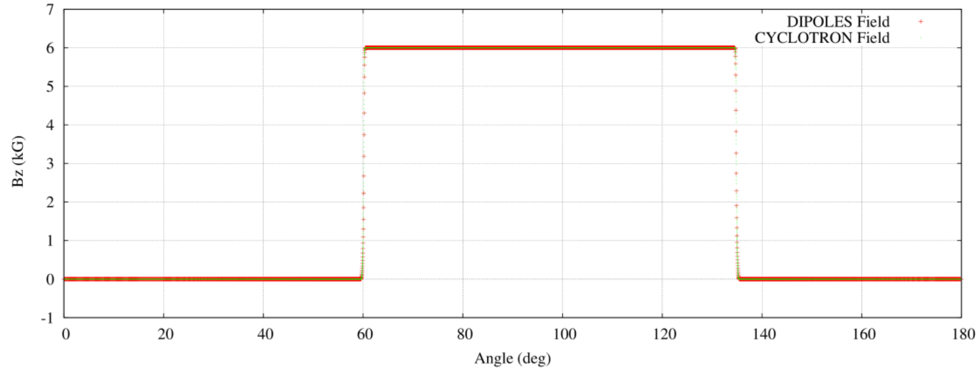
$$\mathrm{g(R)} = \mathrm{g}_0 + \mathrm{g}_1 \times \mathrm{R} + \mathrm{g}_2 \times R^2$$

• The field is then extrapolated off median plane by means of Taylor series: for that the median plane off median plane antisymmetry is assumed and the Maxwell are accommodated.

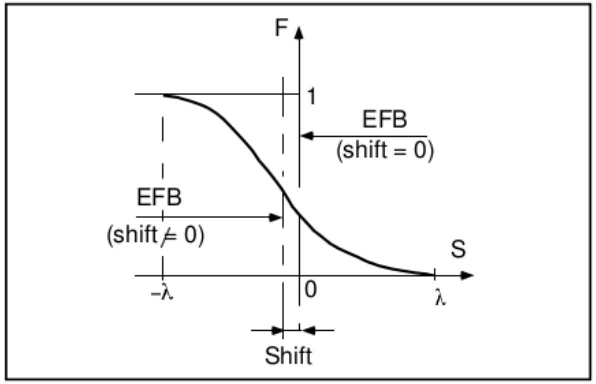
• The EFB can also be straight in which case it is defined by the cartesian equation ax + by + c = 0

"CYCLOTRON": a Dipole magnet





All you need to define is the equation of the field boundary for the magnet: for the radial sector it is a straight line given by: (a,b,c) / ax+by+c=0

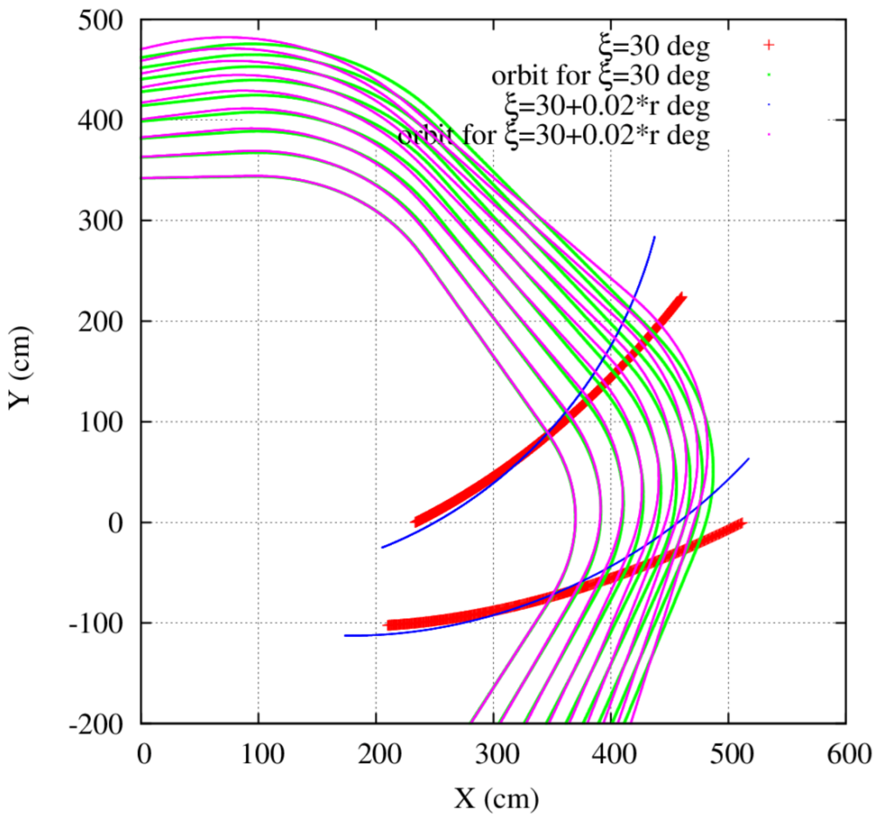


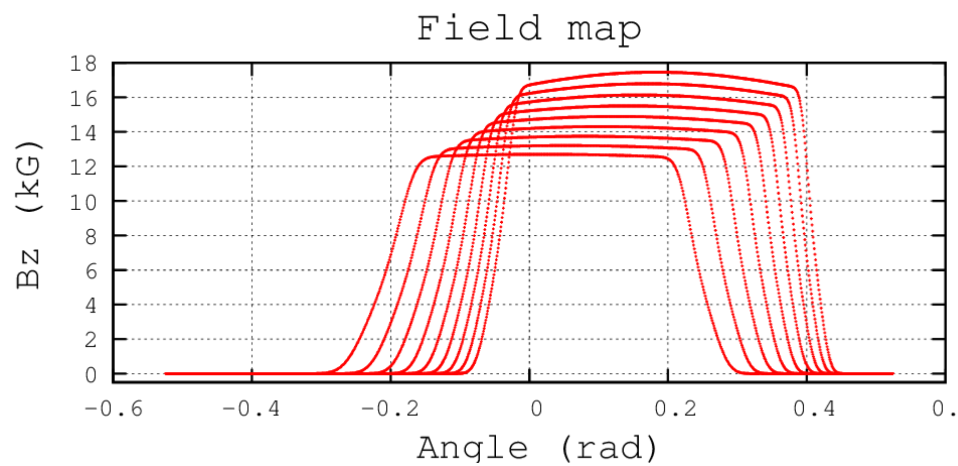
Exponential type fringe field.

“CYCLOTRON” vs “DIPOLES”

Element type	“DIPOLES”		“CYCLOTRON”	
Generated the same analytical model (of homogeneous field inside a non-symmetric dipole magnet) in order to validate the new element “CYCLOTRON” created in zgoubi.	Radius	Angle	Radius	Angle
	361.1185592	-130.8996939	361.1185592	-130.8996949
	397.2288863	-130.8996940	397.2288862	-130.8996947
	433.3393336	-130.8996939	433.3393336	-130.8996945
	469.4498738	-130.8996939	469.4498738	-130.8996943

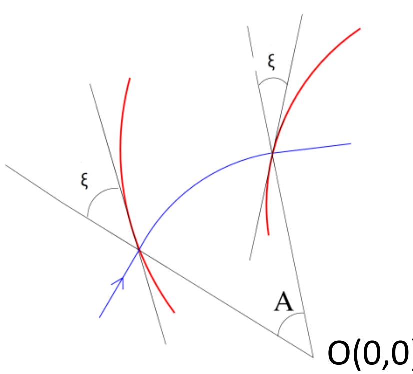
“CYCLOTRON”: a Spiral magnet





Compared with “FFAG-SPI” which has a constant spiral angle ✓: same results

Allow the spiral angle to increase outwards:



$$r = r_0 \exp\left(\frac{\theta}{\tan \xi(r)}\right)$$

ξ is allowed to increase outwards (contrarily to a logarithmic spiral where ξ=cst all over the radius).

“CYCLOTRON” vs “FFAG-SPI”

Element type	**“FFAG-SPI”**		**“CYCLOTRON”**	
Generated the same analytical model (of homogeneous field inside a spiral magnet, ξ=30 deg) in order to validate the new element “CYCLOTRON” created in zgoubi.	Radius	Angle	Radius	Angle
	436.4134359	-153.0288522	436.4134296	-153.0288433
	484.9023805	-208.0583427	484.9023747	-208.0583355
	535.3352512	-258.2960949	535.3352558	-258.2960890
	561.2916958	-281.8654176	561.2916905	-281.8654123

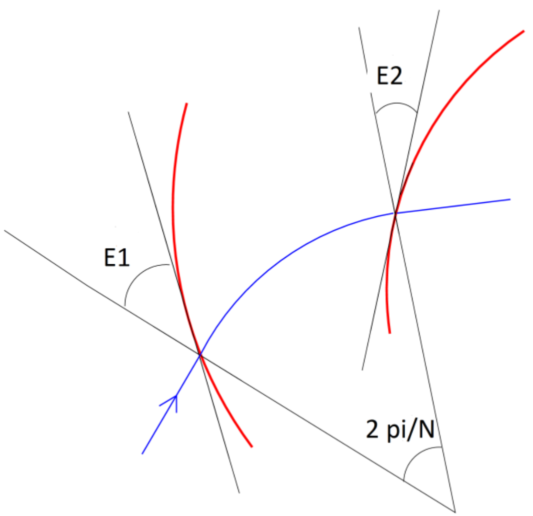
Motivation:

Based on the hard edge approximation and some geometrical considerations, it can be shown for the edge angles that:

$$E1 = \frac{\pi}{N} - \frac{\alpha}{2} + \xi 1$$

$$E2 = -\frac{\pi}{N} + \frac{\alpha}{2} + \xi 2$$

where α is the width of the magnet



Orbit section in one magnet sector: the entrance angle E1 and exit angle E2 are defined positive in the situation shown here.

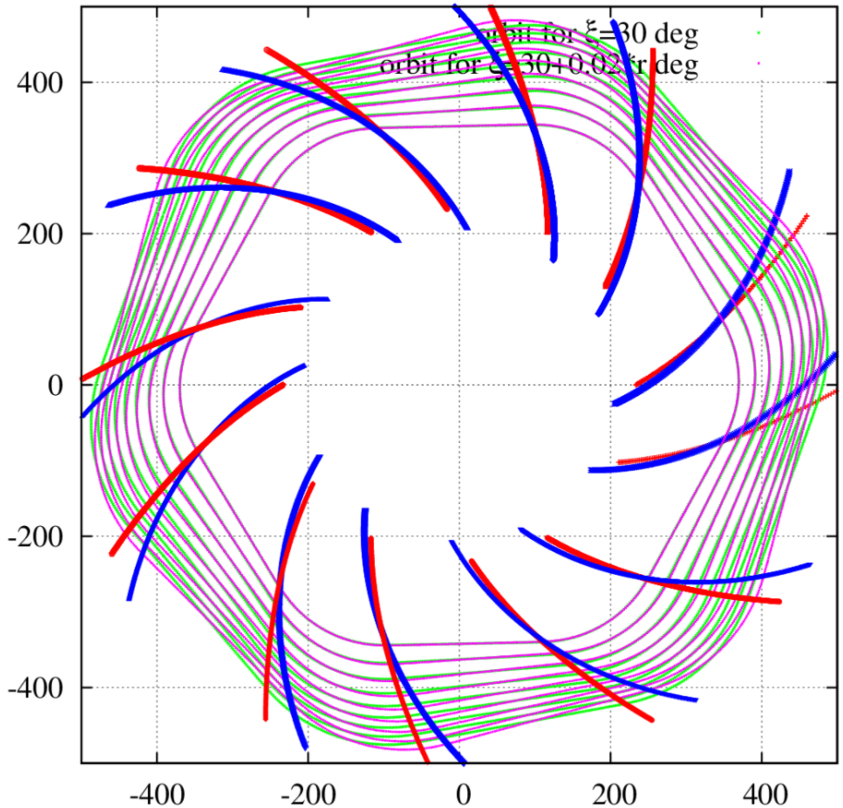
The expression of the horizontal and vertical tunes follow from that, and by imposing the condition of isochronism, one obtains:

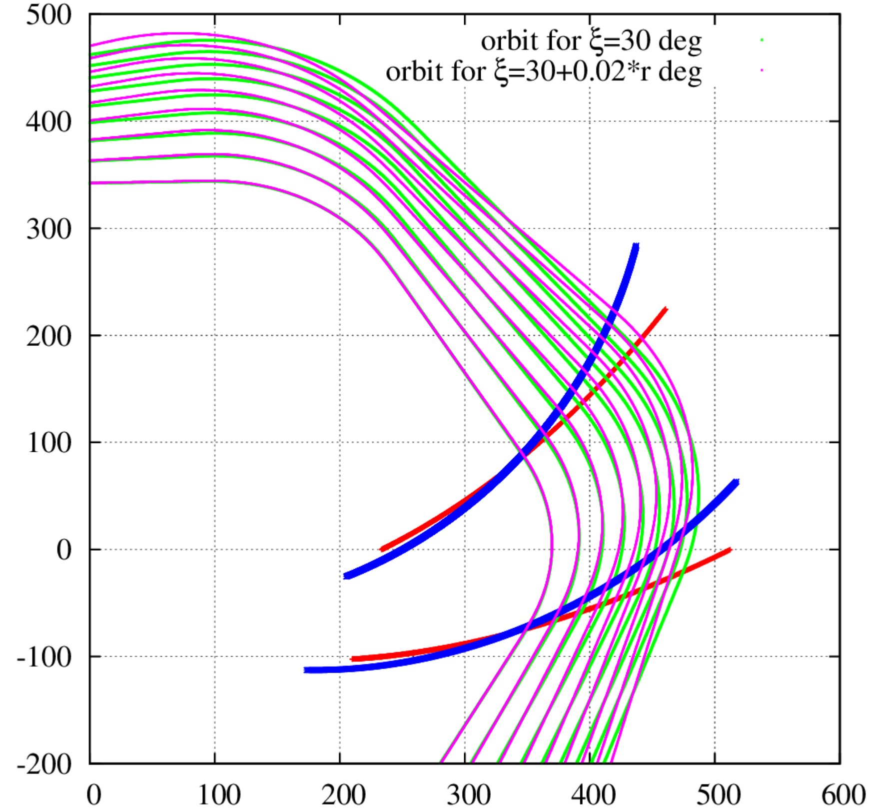
$$\nu_z^2 \approx -\beta^2\gamma^2 + F^2(1 + 2\tan(\xi)^2)$$

From a prescribed value of ν_z, it would be possible to determine the spiral angles ξ1 and ξ2 and hence the magnet shape.

$$F^2 = \left\langle \left(\frac{B}{\langle B \rangle} - 1 \right)^2 \right\rangle$$

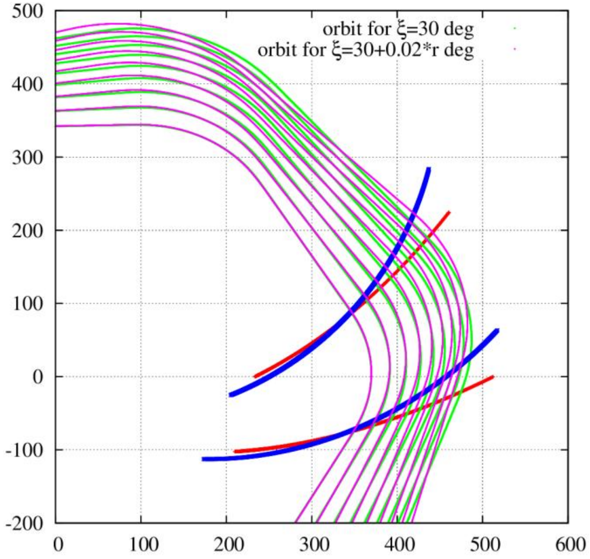
“CYCLOTRON”: a Spiral magnet

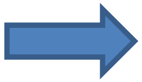


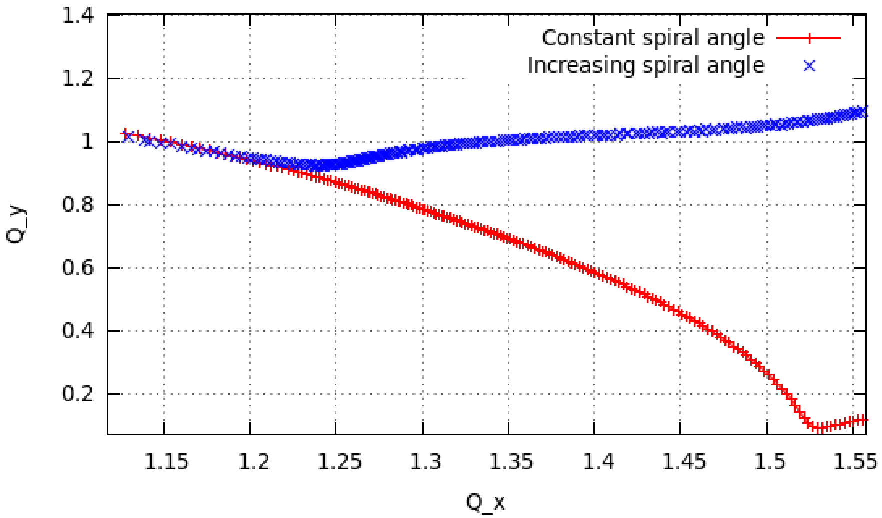


Plot of the closed orbits of a 6-sector cyclotron machine for 2 different configurations: case1 (EFB in red): it has a constant spiral angle ξ=30deg; case2 (EFB in blue): the spiral angle increase linearly as a function of R.

“CYCLOTRON”: a Spiral magnet







Comparison of the tunes.

Application to the case of the PSI ring cyclotron (1/4)

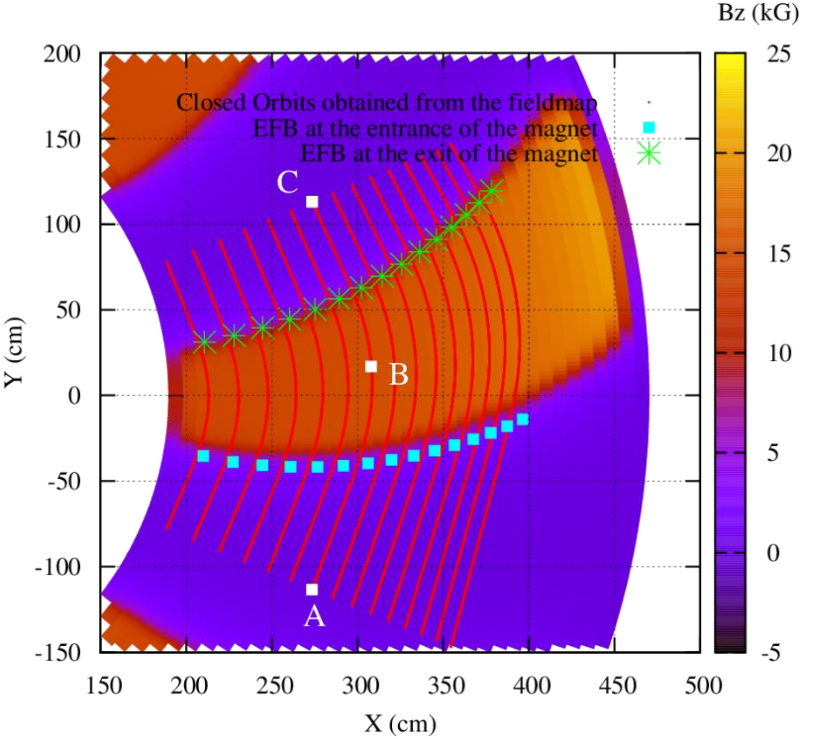
- Obtain the equation of the EFB for the PSI ring cyclotron:

- Effective field length definition:

$$Leff_{ent} = \int_A^B \frac{B_z dl}{B_z^{max}}$$

$$Leff_{exit} = \int_B^C \frac{B_z dl}{B_z^{max}}$$

- By fitting the data points at the entrance and the exit of the magnet to the equation of a spiral, one obtain



EFBs as obtained from the field calculation along the different closed orbits

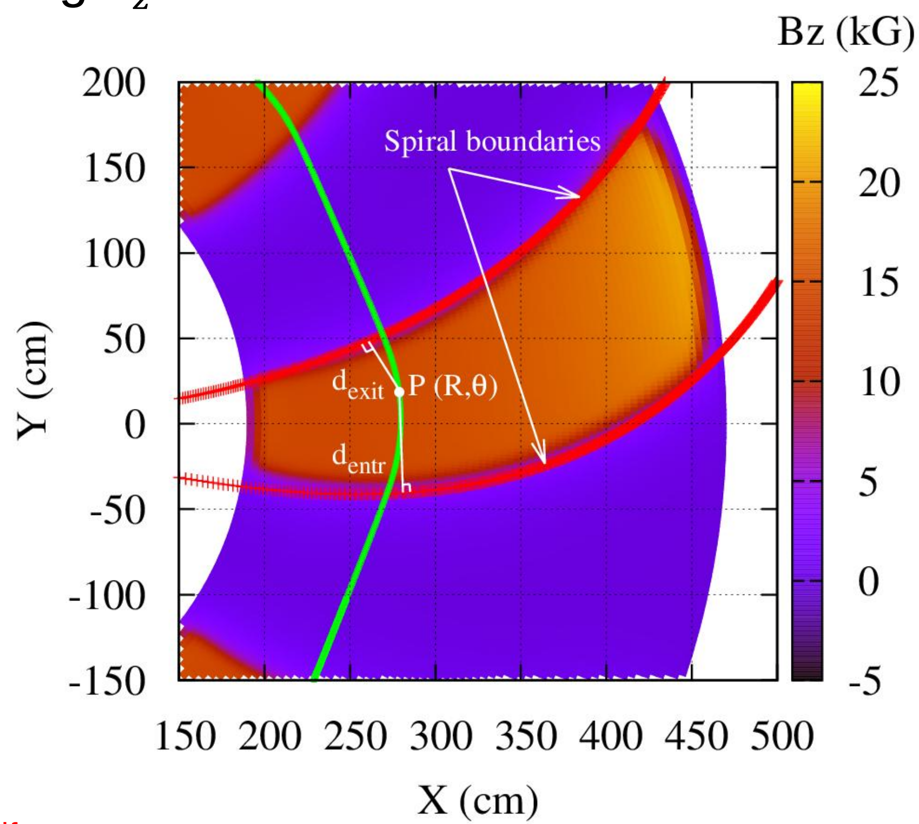
Application to the case of the PSI ring cyclotron (2/4)

- The radial field law is obtained by fitting B_z^{max} for the different closed orbits as a function of the radius.

- The polynomial coefficients $(C_0, \dots, C_5)$ used to determine the fringe field coefficient $\mathcal{F}(R, \theta)$ are obtained using a fitting method. A single closed orbit is chosen for that.

- A full description of the method is provided in

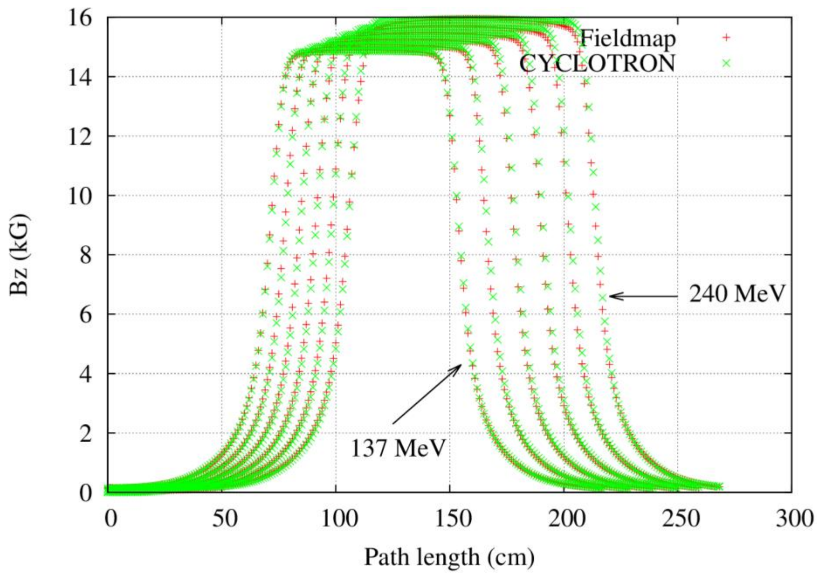
http://www.bnl.gov/ffag14/tutorial/ZGOUBI_DOCS/cyclo_model.pdf

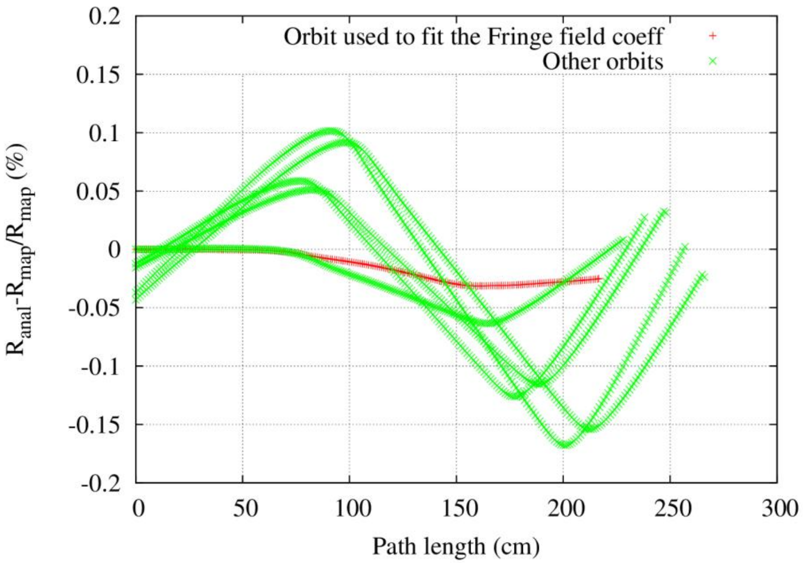


Orbit trajectory as obtained from "CYCLOTRON" model with some geometrical parameters used for the tracking.

Application to the case of the PSI ring cyclotron (3/4)

- The results are shown below:

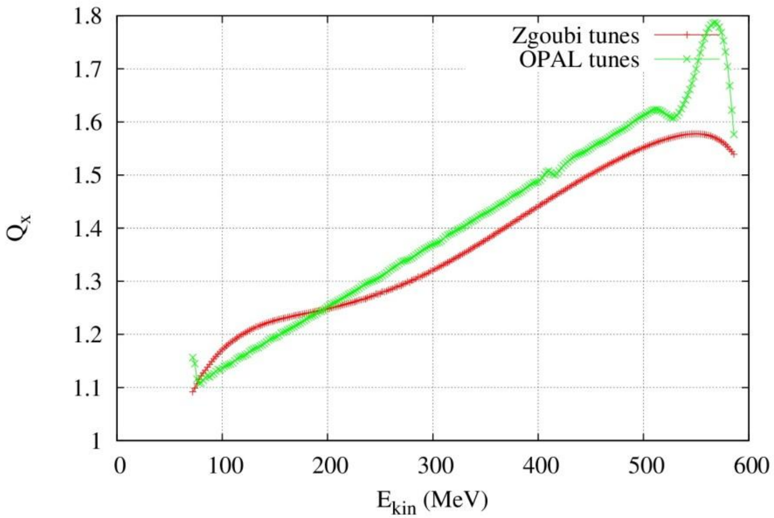


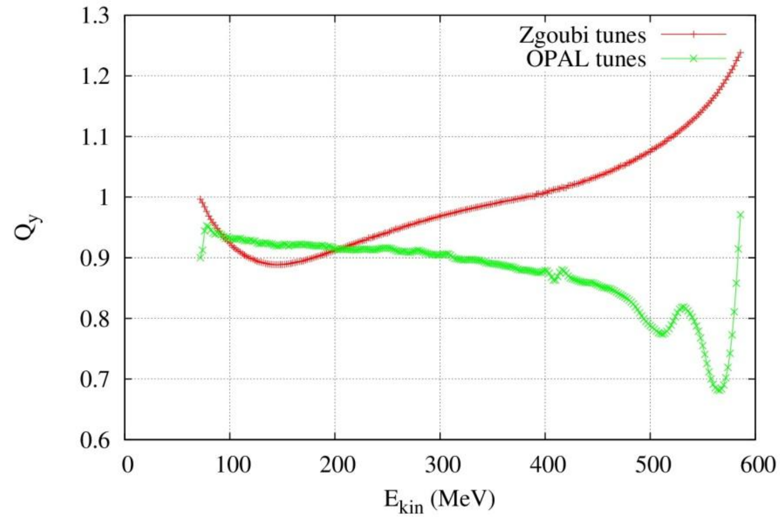


Comparison of the magnetic field along different trajectories (137, 156, 176, 196, 218 and 240 MeV) obtained from the tracking using the analytical model "CYCLOTRON" and using the Fieldmap ("POLARMES").

Comparison of the trajectories obtained from the tracking using the analytical model "CYCLOTRON" and from the fieldmap using "POLARMES": the relative error is less than 0.03% for the 137MeV oribt while for higher energies, up to 0.2 % relative error is obtained.

Application to the case of the PSI ring cyclotron (4/4)





Comparison of the radial and axial tune obtained with both methods.
The fringe field coefficients should be allowed to increase as function of R to improve these results.

Acknowledgements:

For all the fruitful discussions that we had so far:

M. Bai, S. Berg, M. Blaskiewicz, N. Brown, D. Brown, M. Herman, W. Fischer, B. Horak, F. Meot, S. Peggs, T. Roser, N. Simmos, M. Todosow, D. Trbojevic, N. Tsoupas, B. Weng and all!

The contributions of our colleague Hans Ludewig (deceased) are gratefully acknowledged!


Nuclear-The Foundation of Clean Energy
Thank you

Backup Slides:

The ν_s neutrons injected in the core (per incident proton) are successively multiplied by k_{eff}. The total number of neutrons per incident proton is thus:

$$N_t = \nu_s \left(1 + \ k_{eff} + k_{eff}^2 + k_{eff}^3 + \cdots\right) = \frac{\nu_s}{1 - k_{eff}}$$

Among these N_t neutrons $N_t - \nu_s$ are produced by fission.
Each fission produces ν neutrons, thus the total number of fissions per incident proton is:

$$N_f = \frac{N_t - \nu_s}{\nu} = \frac{\nu_s}{\nu} \frac{\mathrm{k}_{eff}}{1 - k_{eff}}$$

And the thermal power of the core is:

$$P_{th}(MW) = E_f(MeV) \times I(A) \times \frac{\nu_s}{\nu} \times \frac{k_{eff}}{1 - k_{eff}}$$